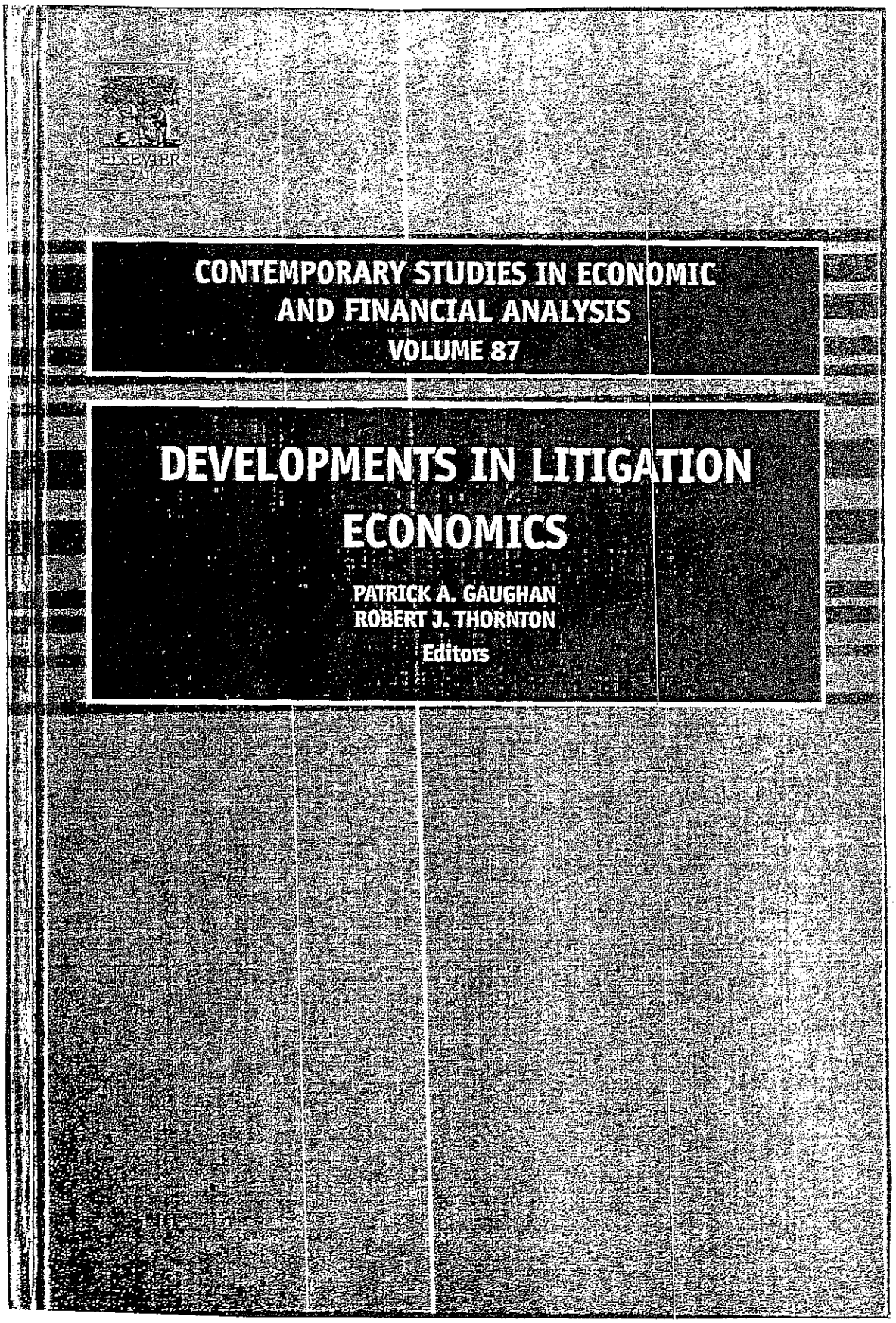
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PATRICK A. GAUGHAN
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Shleifer, A., & Vishny, R. (1997). A survey of corporate governance. *Journal of Finance, LII*(2), 737–783.
Smart, S. B., Megginson, W., & Gitman, L. J. (2004). *Corporate finance*. New York: South Western Thompson.
Smith, C., & Warner, J. (1979). On financial contracting: An analysis of bond covenants. *Journal of Financial Economics, 7*, 117–161.
State Farm Mutual Automobile Insurance Co. v. Campbell et al. (2003). No. 01-1289.
Stiglitz, J. E. (1997). *Economics*. New York: Norton & Company.
Strohmeyer, J. (1994). *Crisis in Bethlehem: Big Steel's struggle to survive*. Pittsburgh: University of Pittsburgh Press.
Stuart, C. C. (1994). Mean stupid defendants jarring our constitutional sensibilities. *California Western Law Review, 30*, 313–345.
Sud, N. (2005). Punitive damages: Achieving fairness and consistency after *State Farm v. Campbell*. *Defense Counsel Journal, 72*(1), 67–79.
Susan Miles et al. v. Philip Morris Companies, Inc., Illinois Circuit Court, (2003).
Taylor, J. (2004). *Economics* (4th ed.). New York: Houghton Mifflin Company.
TXO Prod. Corp. v. Alliance Resources Corp. (1993). 509 U.S. 443, 475.
Vasbinder v. Scott. (1992). 976 F.2d 118 (2nd Cir.).
Viscusi, W. K. (2001). Jurors, judges and the mistreatment of risk in the courtroom. *Journal of Legal Studies, 30*, 107.
Weygandt, J. J., Kelso, D. E., & Kimmel, P. D. (2003). *Financial accounting*. New York: Wiley.
Williams v. Kaufman County. (2003). 343 F. 3d 689, 711 n. 75 (5th Cir.).
Zazu Designs v. L'Oreal, S. A., 979 F.2d 499; 1992 U.S. App. Lexis 28369; 24 U.S.P.Q.2D (BNA) 1828.

FURTHER READING

AOL Time Warner Annual Report. (2003).
Bentham, J. (1881). *Theory of legislation*.
Brigham, E. F., & Houston, J. F. (2004). *Fundamentals of Financial Management* (10th ed., pp. 78–79) New York: South-Western.
Bullock v. Philip Morris Inc. Superior Court of California. (2002).
Klughiet, M. (2002). Where the rubber meets the road: Theoretical justifications versus practical outcomes in punitive damages litigation. *Syracuse Law Review, 52*, 803–846.
Morris, C. (1989). Punitive damages in tort cases. *Harvard Law Review, 44*(8), 1173–1209.
Parkin, M. (2005). *Economics* (7th ed.). New York: Addison Wesley.
Weston, J. F., & Copeland, T. E. (1992). *Managerial finance* (9th ed.). Fort Worth, Texas: Dryden.
White v. Ford Motor Company et al. (2002). U.S. App. Lexis 24364.
Wyeth Annual Report. (2003).
Yahoo Finance. http://finance.yahoo.com. June 1, 2003.

NEW DEVELOPMENTS IN BUSINESS VALUATION

Patrick L. Anderson

1. INTRODUCTION

The purpose of this chapter is to outline new methodological developments in business valuation, with particular attention to how those developments are being used in litigation involving lost profits and the value of operating businesses. In addition to methodological developments, the chapter also includes a discussion of recent legal developments, particularly selected cases that affect the use and standards for business valuation techniques within litigation settings. Finally, the chapter includes a mathematical appendix.

The chapter is broken down as follows:

1. Introduction.
2. Review of standard approaches and sources for those approaches, including newer treatises on methods in business evaluation.
3. Weaknesses in the standard approach, including: critiques of the use of "historical cost"; recognition of the failure of the net-present-value rule; problems with common use of capitalized income, Capital Asset Pricing Model (CAPM) models, and typical rules of thumb; and the support or lack of support for earnings estimates used in practice.
4. Methodological developments in valuation technique, including: "real option" methods; recognition of the option to wait; conditions when the net present value rule is wrong; quantitative methods used in standard

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approaches, such as iterative methods for estimating the cost of capital; and the new dynamic programming valuation method.
5. Legal developments, including developments in allowable methods; the abandonment of the "excess earnings" method; and new case law.

As this chapter is designed to capture new developments, it will invariably suffer from two limitations:

- First, new developments are, by definition, not those that have stood the test of time. Thus, some of the items discussed will be of passing interest, while others will assume increasing importance in the years to come.
- Second, new methods will not have standard nomenclature, nor will they be well documented when compared with their antecedents.

Furthermore, any selection of important "new" ideas relies heavily on the subjective opinions of the reviewer. I have endeavored to capture new techniques and developments with an eye toward identifying the most important and potentially broadly applicable. Invariably, some have been missed, and others will seem much more important to one observer than another.

2. STANDARD APPROACHES

2.1. Generally Accepted Three Approaches

As this chapter is written, one can still say there are "three generally accepted approaches to valuing a business." These approaches are typically defined as:

1. the *market approach*, based on the market value of similar firms;
2. the *asset approach*, based on the value of the various assets that make up a company; and
3. the *income approach* in which an estimated value is calculated by discounting expected future returns. This is sometimes loosely called the "discounted cash flow" (DCF) approach.

While these are still the "generally accepted" approaches, cracks are starting to develop in this easy categorization. In particular, commonly used methods in both the asset approach and the income approach are vulnerable to serious, fundamental criticisms. We will deal with these weaknesses in the section entitled "Weaknesses in the Standard Approaches." In addition, we discuss newer methodologies – some of which call into question the categorization above – in the "Methodological Developments" section.

2.2. References for Standard Approaches

Practitioners in business valuation and litigation economics typically refer to one of a handful of references. To summarize the basic methods, I would characterize these references as largely falling into two styles:

1. those based fundamentally on *accounting methods*, adapted to valuation; and
2. those based fundamentally on *economics methods*, adapted to valuation.

We describe the differences in perspective that appear in these references and cite the important texts for each style below.

2.2.1. Accounting-Based References

The most popular texts in the field of business valuation have, in the past, been written by accountants. These references tend to start with accounting as a basis for business valuation, and then move toward valuing the firm by incorporating techniques from the disciplines of economics or its subdiscipline, finance.[1]

Among these are the following:

1. Shannon Pratt, Reilly, and Schwiess, *Valuing a Business* (1996, 2000).
   Probably the most widely used reference, this comprehensive guide is very heavy on the basic analysis of income statements, cash-flow statements, and other basics of the accounting profession. It contains extensive source notes and excellent practice guides.
2. Aswath Damodaran, *Investment Valuation* (1996, 2002).
   The Damodaran text takes a different tack from that of Pratt et al., by focusing more on finance than on accounting. In particular, his text analyzes capitalization rates extensively and systematically dissects the cost of capital for a firm. The finance approach is based on the venerable CAPM model, which will be discussed later in the "Weaknesses in the Standard Approaches" section. Damodaran's text is supplemented by an extensive website, which contains much useful information on publicly traded firms and historic data on large publicly traded firms.
3. James Hitchner, editor, *Valuation* (2003).
   This very large book contains much secondary material that has been reprinted and collected in one volume. It is light on the derivation of formulas and frequently provides topical "tips" that would be useful for a new practitioner (and good advice for a veteran). However, the different chapters vary significantly in rigor and occasionally duplicate each other.

The extensive reprinted material from other sources, particularly from IRS documents, is quite useful.

4. Jay Abrams, *Quantitative Business Valuation* (2001).

   Abrams ratchets up the mathematical rigor in his text, which derives discount formulas that are often summarized in other texts. In addition, he suggests two innovations that we will discuss further: a log-size model for estimating equity cost of capital, and a method for iteratively calculating the cost of capital for an entire firm. This book does not attempt to comprehensively present valuation techniques, but it is important for its innovation and rigorous treatment of formulas that are often used carelessly.

There are, of course, other texts that are used in this field and which largely follow the perspective taken by these authors.[2]

2.2.2. Economics-Based References

There have been fewer business valuation texts starting from the basis of economics.[3] However, the relative dearth of sources from the economics perspective has been filled by at least two texts that have been recently published. These are:

1. Patrick Gaughan, *Measuring Business Interruption Losses* (2003).

   Gaughan's book begins with an extensive discussion of the source of earnings for companies and describes how those earnings can be estimated. It also describes various legal underpinnings for determining lost profits and explains important concepts in litigation economics that are relevant for any practitioner in the field. The Gaughan text does explain the "generally accepted" three approaches, although it does so in less detail than the accounting-based texts. Instead, he spends more time connecting the concepts used in the valuation calculations with those developed in an economic and industrial analysis. Reflecting the economics perspective shared by books of this style, the analysis of the economy and industry that underlie the forecasted future earnings are prominently and extensively discussed.

   In addition, Gaughan specifically describes the analysis of lost profits for businesses, which is not described at any length in the accounting-based references cited above.

2. Patrick Anderson, *Business Economics and Finance* (2004b).

   This text matches the economics perspective taken by Gaughan. It begins its discussion of business valuation with an entire chapter devoted to modeling the economics of a firm. Anderson notes there are two main

factors that must be determined to value a firm using the capitalized income approach: forecasted future earnings, and a discount rate for those earnings. Anderson argues that properly forecasting earnings is often given short shrift in the valuation literature, and that such omissions lead to systematic errors in valuation. The Anderson text is also unique in that it describes two topics that are generally missing from other references: the role of uncertainty in forecasted cash flows, and the dynamic programming valuation method. These are both described later in this chapter under "Methodological Developments."

2.3. Reconciling the Styles

It is worth noting the differences in perspective among the authors noted above. However, one should not overstate them. In particular, we note that all the "economics" texts rely on accounting concepts for their analysis of businesses. Furthermore, the "accounting" texts recognize the need to adjust accounting records toward economic income. For example, Pratt et al., (1996, p. 151) states:

> It may be worthwhile to define the term economic income, as we will use it in this discussion of the income approach to valuation. As the term implies, we define income according to the economists' definition and not the accountant's definition.

All the texts listed above assume that a spreadsheet software program is used in the analysis. Most texts, particularly the accounting-based ones, almost exclusively describe techniques done in a spreadsheet.[4] This is consistent with the author's informal survey of software use among forensic economists.[5] However, there are a handful of books that extend mathematically beyond what is commonly done in spreadsheets. In particular, the Abrams text describes the use of Visual Basic routines within spreadsheets; the Gaughan text describes certain statistical measures of markets; and the Anderson text describes the use of a mathematical programming environment to create simulation and iterative models.

3. WEAKNESSES IN THE STANDARD APPROACHES

In this section we describe weaknesses that have been identified in the three "generally accepted" standard approaches. The reader will note that these "weaknesses" run the gamut from expected limitations that do not undermine

the fundamental strength of the approach, to severe weaknesses that in many cases mean that the approach is unsuitable. We take these in order.

3.1. The Market Approach

The market approach should be the strongest approach for the valuation of a firm, because it closely matches the definition of "market value": the price agreed to by a willing buyer and a willing seller, neither under any compulsion, and both having adequate information.[6] If there is market evidence available, there is no reason why it should not be given the strongest weight.

However, we then come to grips with a fundamental problem: there is *not* market evidence available for most closely-held companies. This problem is compounded by the fact that *most* companies are closely-held companies. Indeed, despite the voluminous examinations of publicly-held companies and the enormous amount of data on market values for equities in those firms, most companies are not publicly traded, and therefore cannot be easily valued using stock-market data.

3.2. Critique: Inconsistent Reporting of Business Sales

In an attempt to fill this gap, there are a small number of data sources that have been built up from reported sales of closely held companies. These databases provide some indication of how market values have been actually determined for closely held firms. However, a recent analysis by Wolpin (2003) provided ample evidence that these databases should be used with caution. The author identified significant weaknesses in these databases, related to the following:

1. lack of consistency in reporting, with users reporting sales on multiple different bases;
2. nonrandom reporting, which severely limits the usefulness of the data for inferring information about the general population of companies; and
3. selection bias, which the author felt occurred for both the submitters of data and those that record it.

Wolpin performed a series of statistical tests that support his contentions that the commonly used business data should be used with caution. In particular, he argues that the collectors of data have a vested interest in their use and have exaggerated their reliability. He argues that the data should only be considered reliable when they reflect an active market with sufficient observations.

Wolpin does not argue that these databases are useless. Indeed, such an argument would overstretch his findings, which indicate that the data cannot be used to reliably infer information (such as the mean) about the population as a whole. His critique is a useful warning against naively using these databases as if they were a random sample of actual business sales.

3.3. The Asset Approach

The asset approach includes valuation methods that begin with cost information for specific assets within a firm. By the accounting identity, the assets of a firm must equal the sum of liabilities and equity. However, this identity becomes less useful when we speak not of the accounting balance sheet, but of the value of the firm in the market. We discuss below the critiques of the asset approach when applied to an operating business.[7]

3.4. Critique: Historical Costs Do Not Predict Market Value

There is no principle that so neatly summarizes the epistemological difference between accountants and economists than reliance on "historical cost" accounting records in business valuation. This ambivalence in the business valuation community is reflected in the differences in perspective taken by authors of valuation references.

But the question is not merely one of perspective or style; many business valuations begin with an examination of accounting records based on the historical cost principle. Furthermore, some valuation techniques classified as part of the "asset approach" are based on the validity of those records as indicators of market value. Others, at least in practice, rely almost entirely on these records as predictors of the future.

The classic accounting method is built around the principle of historical cost, and indeed could not exist without it. A brief history of accounting reveals how important its development has been for commerce and for the growth of the world economy since the Renaissance.[8] Given that the market value of a firm will fluctuate while its accounts must have some stability, the historical cost principle seems to be the strongest foundation for

reporting the assets, liabilities, and income of a firm. From an accounting perspective, the use of historical costs is a vital principle, which ensures an objective measurement of an actual transaction and is the basis for accounting statements.[9]

On the other hand, economists from Adam Smith onward have debated whether historical costs have any meaning at all.[10] Economists have typically focused on the labor inputs, the scarcity, or "supply and demand" to derive value. Advances in finance over the past several decades have resulted in an extensive theory of the market value of financial assets, based on the avoidance of arbitrage and the use of replicating portfolios.[11] In these settings, historical cost is the basis for an accounting record, not an indicator of value.

However weak the theoretical basis for their use in estimating future market value, the historical cost principle is a powerful and beneficial one when viewing the current financial status of a firm. Indeed, even economics-based valuation texts suggest starting a valuation exercise with an examination of the fundamental accounting records, all of which are based on historical costs. Still, while historical cost accounting records are essential to understanding a business, they are not indicators of market value.

3.5. Critique: The Failure of the "Excess Earnings" Method

Problems arise when historical cost records are used to predict the market value of an operating firm. In general, the "asset" approach of valuation is founded on the notion that historical costs predict, in some sense, the market value of an asset. One business valuation method is firmly based on this notion. Known as the "excess earnings" or "formula" method, it arose from a 1920 U.S. Treasury method of valuing the lost earnings from breweries shut down by Prohibition.[12] The IRS restated it in a 1968 Revenue Ruling, with very specific warnings and restrictions on its use.[13]

The excess earnings method is based on the premise that income above a certain return on tangible assets constitutes "excess" earnings, and therefore is the basis (assuming continuation of those earnings) for the market value of intangible assets. This premise, given the hindsight of over 80 years, should be recognized for its prescience about future developments in portfolio theory and valuation techniques.[14] However, as a valuation technique today, it is deeply flawed.[15] Indeed, the IRS guidance on this from 1968 starts with the admonition that the formula approach should "only be used when there is no better basis available for making the determination."

Neither the flawed assumptions of this method nor its disrepute has stopped its use, however. Valuation estimates using this method continue to be produced. Furthermore, the method is still described – although with significant caveats – in references such as Pratt et al., (1996) and Hitchner (2003).[16] This should change. Recently, the "excess earnings" method has been explicitly and roundly denounced by Anderson (2004b). Gaughan (2003) and Damodaran (1996) do not mention it as a method. The better texts that include it repeat the admonition of the IRS against its use and recount one or more ways it can produce erroneous results. Even if one simply adopts the IRS guidance from 1968 (to use the method only when there is "no better basis available"), it is difficult to conceive of a business that could not be valued with another method. At this point, no practitioner should use an "excess earnings" method, nor should it be presented as a practical method to predict the market value of a firm.

3.6. The Income Approach

The income approach (sometimes loosely called "discounted cash flow")[17] is the workhorse method for many valuations of closely held firms. Without market prices on similar firms, and with assets recorded at historical cost having only a tangential relationship with the firm's value, the capitalized income approach is often the most reliable.

The specific methods used within this approach are well described in many texts.[18] The fundamental tasks under the income approach are forecasting business income (often measured by cash returns to shareholders or to the business enterprise as a whole[19]) and capitalizing it by using a discount rate appropriate for time and risk. Often, the resulting estimate is further adjusted by certain discounts or premia.

We highlight critiques below that reveal newer developments in this field.

3.7. Critique: Inadequate Analysis of Forecasted Income

The first critique has been made forcefully by Anderson (2004b), who notes that many valuation references give short shrift to the essential task of forecasting income. Indeed, it is still common to see valuation estimates that are based on the naive, and almost always incorrect, assumption that the last period's reported earnings will grow at a constant rate forever.[20] Even if the growth rate seems "reasonable," no buyer will invest money based on an

unsupported assumption of perpetual growth. The lack of strong foundation for an earnings forecast is often the weakest point of a valuation estimate done under the income approach.[21] Forecasting earnings requires an analysis of the industry, the economy as a whole, and the company. The economic reasoning required for this is apparently not well taught to many practitioners.

There is strong theoretical and empirical evidence to reject the naive "continued growth forever" assumption. Earnings in any one period depend heavily on business conditions in that period, including business conditions determined by unpredictable events, fluctuations in the economy as a whole, changes in the industry, and unexpected actions by humans within the organization itself. Thus, there is no theoretical reason to expect current-period earnings to grow at a constant rate.

The empirical evidence is almost as conclusive. In a little-noticed analysis published over 40 years ago, Little (1962) found that strong earnings growth in one period was not positively correlated with strong earnings growth in the next. Damodaran (1996, Chapter 7) updated the analysis using data from the 1980s. Again, he found that the correlation coefficient for earnings growth over two periods was not significantly different from zero.

The implication for those forecasting earnings is clear: there is neither theoretical nor empirical justification for assuming that past earnings growth rates will simply continue onward indefinitely. A deeper analysis, involving the industry, economy, and company, must support the forecast. On this point, the IRS guidance is quite clear. Since Ruling Revenue 59-60, issued in 1960, the IRS has explicitly required an examination of the underlying industry, competitive position of the firm, and future earning power of the firm in a valuation analysis.[22] Regardless of the specific method used under the capitalized income approach, the forecasted income must be based on an analysis of the industry and the firm's position in it. Valuations that ignore this step (and simply assuming the trend growth rate will continue *is* ignoring this step) are quite vulnerable to challenge.

The importance of this point may seem obvious, but it has been neglected in practice and not emphasized enough in many reference texts.[23] Recent texts by Gaughan (2004) and Anderson (2004b) place more emphasis on this task and suggest how it should be done.[24] Practitioners today must – if they wish to properly complete a capitalized income valuation estimate – base their earnings estimate on an analysis of the economy and industry. For practitioners with only an accounting background, it may require the participation of an economist on the valuation team.

3.8. Critique: Naive Use of CAPM Model

The CAPM model, even with its many critics, has been the most powerful organizing paradigm in finance for about 40 years. The CAPM model was based on the mean–variance framework for evaluating large portfolios within the universe of equity investments. This approach was pioneered by Harry Markowitz and extended by William Sharpe in the 1950s and 1960s.[25]

The CAPM model describes how, under certain quite limiting circumstances, the equity discount rate for a firm can be determined from a linear combination of a small group of factors: the risk-free rate, the "equity premium" for equities as a whole, and the "beta" factor that relates the earnings volatility of firms in one industry with those of the market as a whole.[26] Together with the leverage of the firm (the share of its market value that is supported by equity), this model seems to provide a straightforward manner of estimating the cost of capital for most firms.

There are compelling critiques of the CAPM model, many of which focus on the benefits of adding additional factors to the model to better predict the discount rate.[27] However, the general principle–that much of the risk and return characteristics in portfolios containing equities of large, publicly traded firms can be captured within the mean–variance framework–continues to be solid.

The discussion above was careful to place the CAPM model within the mean–variance framework, which operates under significant limiting assumptions. Among these assumptions is the availability to the investor of a wide variety of equities, including those that "span" the risk characteristics of the subject firm; good information on these equities; publicly trading in these equities; and minimal transaction costs.

Consider now the use of this framework for privately-held firms. Investors generally cannot purchase shares in such firms. The key data used to analyze publicly-traded firms – earnings, expenses, capital expenditures, etc., – are also generally not available. The shares in the firms are typically not traded very often. There is no traded asset that matches the underlying risks in the firm. Finally, transaction costs are quite high, especially if the due diligence costs are included. In sum, *every one* of the listed assumptions used to derive the CAPM model is violated in most small business valuations. This should cause concern about using the straightforward CAPM model for small firms.

The following example illustrates the problems. There are many sources for "beta" parameters for publicly-traded companies in large industrial segments. While we might question how accurate these are, for this discussion we

assume that they are quite accurate. What does this tell us about a small firm in the same industry? It may tell us very little. For example, there are major alcoholic beverage brewers that are publicly traded and whose business is brewing beer. There are many distributors that take these products and distribute them to retailers. Are these companies in the same "beer" business, and, if so, is a beta calculated for a national brewer indicative of the risks faced by a local distributor?

The answer to the first question is mixed, as the distributor also relies on the general market for beer in general and certain brands, in particular. However, the distributor is primarily in a distribution business, not a beer business. It may also have other lines (wine, soft drinks, other brands of beer) and can prosper based on good local service even when its brands are not growing.

The answer to the second question is then clear: the brewery's risk is quite different from that of the distributor.[28] Indeed, it may be that the most comparable publicly traded businesses are in the distribution business, or the retail business, or are similarly dependent on a local area's economy. Therefore, the beta for a big brewer cannot be used to calculate the cost of capital of a local distributor.

The generalization of this example is simple: the "beta" parameters calculated for large firms are often unreliable estimators of the risk in a small firm, even if that firm is in the same broad industry. Without a reliable beta estimate, the CAPM technique of estimating discount rates falls apart. This does not mean that the capitalized income approach does not work, but it does mean that a CAPM-based technique for estimating the discount rate does not work. This critique also applies to "build-up" discount rate estimates that are based on CAPM model.

3.9. Critique: Excessive Use of Discounts and Premia

A significant literature has arisen describing various discounts and premia that should be used when adjusting a capitalized income estimate of the value of a firm. In many cases, these discounts or premia provide a useful and necessary adjustment for factors that are outside the standard discount rate analysis. The workhorse capitalized income approach typically produces a preliminary value estimate that should be further refined through the use of discounts and premia. These adjustments should reflect the particular characteristics of the company or the type of equity investment allowed in the company.

However, a review of the use of the adjustments would observe the breadth, severity, and ubiquity of such factors. This observation motivates the following critique of the excessive and improper use.

First, consider the range of discounts and premia:

1. One well-developed concept is the discount for marketability on stock or other equity that is burdened by resale restrictions. Basic economics would indicate that the ownership of an asset implies the right to sell it; restricting that right should reduce the asset's value. A series of empirical analyses have demonstrated convincingly that restrictions on resale cause securities to be priced less than identical securities without the restrictions. Thus, both theory and practice coincide on this concept.
2. Other discounts and premia have been proposed and used for contingent liabilities, minority interests, "blockage," control, size, nonhomogeneous assets, and other factors.[29] Indeed, there is now at least one book devoted entirely to discussing discounts and premia.[30]
3. Some discount factors are applied to the firm as a whole; some to particular interests in the firm; and some to the cost of capital for the firm.

With this range in mind, we now critique the excessive use of discounts and premia in three categories, and then conclude with two warnings for practitioners.

Some of the "discounts" in the second category above appear to extend beyond an adjustment to a well-established value, and are instead a ratio of that value to that of a *fundamentally different entity*. In particular:

- A company with significant contingent liabilities or assets is *not* the same as a company without them. A "discount" or "premium" should be an adjustment to the value of the particular entity, not the difference between the value of one company and another. An analogy will illustrate this: the cost of a Chevrolet sedan will typically be smaller than that of a Cadillac sedan. The difference in price is not a "discount" off the price of the Cadillac – it is a lower price for a less valuable vehicle.
- A contingent liability should be valued separately and recognized as such. Calling the reduction in value due to contingent liabilities a "discount" is at least mislabeling; it may be a gross error. A significant contingent asset – say a patent or license – is not cause for a premium over the discounted cash flow value–implied value. It is an additional source of future cash flow and should be valued separately.
- Nonhomogeneous assets may need to be valued separately, and considered as separate entities.[31] This is particularly the case if the entity as a

whole is going to be sold. A simple "nonhomogeneous" discount will probably not capture the costs, or benefits, of this step.
- A small-sized company is often a qualitatively different entity than a large-sized entity. This difference can arise from access to capital, dominance of one or more key persons, the form of organization, and the ability to offer publicly traded securities.[32] These differences imply changes in other aspects of the valuation analysis rather than a simple "size discount."

Furthermore, some of the discounts are based on similar factors. The use of two discount factors – both related to the same underlying factor – is double counting, or "double discounting."[33] For example:

- A historical analysis underlying a size premium or discount may capture factors such as discounts for marketability.
- Studies of discounts for marketability probably include companies with controlling interests and other factors.

Thus, one cannot apply two or more discount factors that account for the same underlying risk.

Many common methods of estimating the cost of capital rely on discounts and premia. Practitioners using these approaches should consider the following:

- Historical cost-of-capital data are usually derived from large, publicly-traded firms. Cost-of-capital estimates for firms that match the characteristics of these data should be subjected to minimal adjustments; firms that do not should be adjusted once for each characteristic that differs.
- In particular, CAPM models that use "beta" factors to estimate the cost of capital of specific firms *already have adjustments* for the typical firms in the sample for these industries.

3.10. Warnings for Practitioners

Practitioners should keep in mind the following cautions when applying discounts or premia to the discount rate or the underlying cash flows:

1. The indiscriminate use of premia and discounts should be recognized, discouraged, and criticized. The use of "double discounts" or simple application of "average" discounts should be classified as an error and avoided. This critique has recently been made by other authors.[34]

2. The use of discounts and premia when estimating the cost of capital should be done with caution when a subsequent discount is used on the overall valuation estimate. In many such cases, only one discount factor should be employed.
3. When the entity being valued has fundamentally different characteristics – such as contingent liabilities or assets, nonhomogeneous assets, or an unusual capital structure – than a standard model incorporates, the practitioner should *use a different model.* A discount or premium applied to an estimate produced by a standard model will generally be incorrect.

3.11. Implication: Weakness in the Capitalized Income Method

There is a more fundamental implication of this critique. Even in cases where discounts are properly applied, it appears that they can easily range upward of 50% of the amount of a straightforward capitalized income value estimate. This could mean two things: adjustments are big in this world, or the fundamental method is broken. The lack of good data on actual market transactions both creates the need for methods like capitalized income and limits the ability to test it. However, if we assume that the limited data available imply that discounts or premia routinely exceed 25% of the estimate derived using a standard method, we should ask ourselves the following question: Is the standard estimation method sound?

At this point, the evidence from good valuation practice indicates that the standard method, well applied, is sound. However, we should expect that the next decade will result in the question being posed more forcefully.

4. METHODOLOGICAL DEVELOPMENTS

4.1. Log-Size Discount Model

Jay Abrams has proposed a simple and robust model for estimating discount rates for firms that fit into the small- and medium-sized categories. This "log-size" model uses a simple logarithm of the size of the assets of the firm to arrive at a discount rate. The model is described well in Abrams (2001).

Abrams provides empirical evidence to support the method. Aside from Abrams' data on the log-size model, there is ample evidence of a "size

effect" on the returns of private firms. This shows up in tests of the CAPM model, and in straightforward analyses of the cost of capital.[35]

There is no rigorous theoretical construct from which the log-size model can be derived. Therefore, if the assumptions underlying a CAPM or other model are fulfilled in a certain case, it should be preferred over another approach lacking theoretical support. However, the critiques of the CAPM are serious, especially for small firms and those firms not represented in the universe of publicly traded companies. Therefore, practitioners should consider whether a simpler, more robust model deserves precedence in such cases.

4.2. Iterative Cost-of-Capital Model

A common error in valuations under the capitalized income approach stems from a misuse of the historical cost–balance sheet when weighting the debt and equity parts of the capital structure. When calculating a weighted average cost of capital (WACC), many practitioners commonly use as weights the book value of debt and equity. The actual weights should be the *market value* of debt and equity. For most operating firms, the market value of equity will typically be a much higher share of the total capitalization than the book value. Reading a balance sheet for a firm, based on historical cost accounting, and using those figures to weight the cost of capital will often result in a *substantial overestimate* of the market value of a firm.[36]

While one source of this error is simple ignorance, the other is unfamiliarity with iterative methods to solve the mathematical problem of estimating one variable (value) when an input to that calculation (cost of capital) depends partially on the variable one is trying to estimate.

Three recent texts (noted below) describe a practical iterative method to solve the mathematical problem:

1. First, use the book value to prepare a first guess of market value and use these weights to prepare a first guess of WACC;
2. Use this initial WACC estimate to, in turn, prepare a second guess of market value;
3. Use the second guess of market value to prepare the second guess of WACC; and
4. Continue until the results are close enough to warrant no further iterations.

These iterations are not difficult to perform manually on a standard spreadsheet, and Abrams (2001, Chapter 6) describes such a method. The entire process can be programmed into a more sophisticated mathematical model, which is described by Anderson (2004b, Chapter 11). Pratt (2002, Chapter 7) also describes the iterative method and highlights how failure to properly weight the capital structure can produce serious errors in value estimates. A related application is presented in Anderson (2004b, Chapter 10), in which an iterative method is used to properly estimate the value of income-producing real estate.

4.3. Investment Under Uncertainty

One of the most powerful critiques in recent decades, and one that is only now surfacing in the standard texts,[37] is a direct attack on one of the most hallowed rules of finance: the net present value rule of investment.

The net present value rule can be stated as follows: if the expected net present value of the cash flows from an investment exceed the amount required to make the investment today, one should make the investment. There is no trick in this statement; the investor (or manager of a company) should, according to the rule, estimate the future returns, taking into account the relative probability of future events, and discount them. The resulting expected NPV should be compared with the amount of the investment. If the expected NPV is greater, the investor should go ahead with the transaction.

This rule will, if funds are sufficient to outlast random fluctuations, make one rich in a casino.[38] In the business world, following the NPV rule has long been taught as the foundation for managers attempting to maximize shareholder value.[39] Consider the standard NPV rule used in most investment texts, such as the following:

> Theory of Valuation
>
> ...the value of an asset is the present value of the expected returns.[40]

This is echoed in corporate finance texts:

> Value today always equals future cash flow discounted at the opportunity cost of capital.[41]

The essence of the attack on this rule is that it ignores a powerful source of value: the *option to wait*. Indeed, the option to wait and make the investment

later is an essential part of actual markets. Ignoring a consideration used by most investors is theoretically unsound. In other words, *the standard NPV investment rule is wrong.*

This critique was well developed in the seminal book by Dixit and Pindyck (1994), *Investment Under Uncertainty.* They summarize previous research conclusively showing that the option value in many investment management decisions was significant enough to cause decisions made with the straightforward NPV rule to lose money by comparison. Such examples have also been used in a handful of other economics and finance books.[42] However, as cited above, the NPV investment rule is still enshrined in many investment and valuation texts.

Most human analysts understand the natural human tendency to wait until the time is ripe to make the deal. As will be discussed in the following sections, there are now a number of analytical tools that can be used to make such judgments explicit.

4.4. The "Real Options" Method

The explosive growth of the market for derivative securities has pushed – and been pushed in return – by the development of analytical tools to estimate the value of financial options. Since the publication of the seminal paper identifying the Black–Scholes option valuation formula in the 1970s, a number of innovative financial option models have been developed.[43] These include various binomial tree models, Monte Carlo methods, and variations on the classic Black–Scholes model.[44]

4.4.1. Valuing the Equity as an Option

One of the most intriguing insights emerging from the original Black and Scholes article was the observation that the equity in a corporation could be viewed as a call option on the value of the firm, with the strike price being the value of the debt. They noted that equity holders have the residual claim on the firm's assets. If the firm is liquidated, the proceeds will first pay off all bondholders, and anything left will go to the equity holders.

For going concern firms, stocks are equivalent to "in the money" call options on the value of the firm. These securities can be sold and re-sold indefinitely, as long as the market perceives the firm's value as higher than its debt.

4.4.2. The "Real" Option

In the past decade, a number of authors have developed a "contingent claim" approach to valuing firms, based on this insight.[45] Because equity in a firm can be viewed as a call option on "real" assets (the firm being an actual, rather than a financial, asset), this approach is sometimes called "real options." Real option methods are particularly suited in the following situations:

1. When valuing firms with significant contingent assets or liabilities. In such cases, contingent claims approaches are obviously better suited to the underlying asset or liability.
2. When firms are in financial distress, or the "going concern" assumption is not warranted or is questionable.
3. When firms have particularly promising technologies or intellectual property such as patents or licenses, which could result in significant cash flows in the future.

Real option methods are not as well developed as most other valuation techniques. Furthermore, they often rely on standard approaches (such as capitalized income) to value the expected earnings of operating entities. Option methods are then used to value contingent claims.

Note that the key "risk" involved in option methods is not the same as "equity risk," or even equity risk adjusted for the "beta" of a particular industry.[46] The key measure of risk in standard option valuation formulas is the volatility of the underlying asset, on which a contingent claim exists. The more volatile such an asset, the more likely it will end up "in the money" even if it is currently "out of the money."[47] Note that the traditional implication of volatility in returns is a higher rate of return, meaning a lower present value. In the standard analysis of an out-of-the-money call option, more volatility implies *higher* value. This difference in the effect of volatility on market value is not, when properly considered, a complete contradiction.[48]

We do not describe real option methods here. Damodaran (1996, 2002) provides the most extensive treatment among the standard references. Hitchner (2003) contains a very brief summary. Anderson (2004b) describes the approach as part of an overall presentation on risk and the options available to managers and shareholders, and also introduces the related Dynamic Programming method.[49]

Like all other valid methods, a properly completed option-method valuation, given good information, will provide a similar estimate of fair market value. However, at this point the real options approach has not been widely used in practice and would benefit from more development.

4.5. Modeling Risk: Uncertainty in Future Earnings

Forecasting earnings is the primary task in a capitalized income approach. Typically, such earnings are forecasted based on past performance. The critique above describes weaknesses in this practice, due to inadequate (or nonexistent) review of the underlying economic and industrial conditions.[50] We consider below methodological innovations in the treatment of *risk* in earnings, assuming that the underlying economic and industrial analysis has been properly completed.

4.6. Complex Uncertainty in Future Earnings

Most valuation texts discuss uncertainty primarily with respect to the cost of capital. More volatile earnings in the past are associated with higher expected returns, and therefore a higher discount rate on future earnings. However, the uncertainty in other business variables should be considered as even more important. In particular, the uncertainty in expected *future* revenue and earnings is more important to most business valuation exercises than the analysis of past earnings rates. Anderson (2004b) describes techniques for modeling different types of uncertainty in cash flows, including:

1. Simple deviation around a trend. This appears to be the most common modeling of uncertainty in valuation exercises; most examples presented in the standard texts show forecasted earnings following a very strong trend, often with no deviation around it.[51]
2. Complex uncertainty, involving both a drift and a random variation around the drift. The well-known "random walk" motion is an example of a process with such uncertainty. A more developed model is known as "geometric Brownian motion," and appears to well represent stock prices.
3. Complex uncertainty involving "jump" processes, which we discuss in the next section.

Dealing with uncertainty is a fundamental concern in financial economics. Mathematical techniques such as stochastic calculus are now commonly employed to model asset prices in the academic literature.[52] While some of these techniques are more esoteric than needed for applied work, there is an unsettling gulf between the treatment of uncertainty in most valuation texts and its treatment in both the academic literature and in the financial markets.[53]

Anderson (2004b) presents a set of tools to model these kinds of uncertain cash flows. We discuss one such innovation with broad application in business valuation below.

4.7. "Jump" Processes and Risks of Termination

Most treatments of risk in earnings are based on the assumption that periodic returns are distributed normally (or at least symmetrically) around a mean return. The distribution of returns can then be largely described by two "moments" of the distribution: the mean, and the variance.[54] This is the basis of the "mean–variance" framework for analyzing investment portfolios, on which the familiar CAPM model and its many variants are based. Using this assumption, revenue, earnings, and other important business variables are often modeled as a trend line subject to symmetric, normally distributed random disturbances.[55] However, there are a certain class of variables subject to risks that are *not* well modeled by random deviation around a trend random walks, or even by geometric Brownian motion. In particular, some variables are subject to sudden, unexpected changes that may dramatically alter the prospects of a business. This type of behavior is often considered a "jump process," because when graphed the line appears to "jump" at one point.

The risks borne by insurance companies are an obvious example of such risks. The chance that any one building will burn down is small, but the number of buildings they insure is large. Nonfinancial companies also bear such risks. For example, companies that manufacture or distribute products under a franchise agreement, or sell predominantly "brand name" products, bear the risk that the brand itself will decay, the products will no longer be produced, or that they will lose franchise rights. Investors bear a default risk that may be small, but is not negligible, for many classes of bonds. Almost all parties to contracts bear a risk of nonperformance by other parties. Businesses with operations in foreign countries bear risk of war, closure of borders, or confiscation.[56] Some important business variables change infrequently, but when changed cause either catastrophic losses or large gains. These cannot be assumed to behave like one of many, individually small, random occurrences that are adequately modeled in a mean–variance framework.

Anderson (2004a) derives a formula incorporating risks modeled by a "jump" process into a discounted present-value calculation. The formula is based on the Poisson distribution, which is described in the "Mathematical Appendix" of this chapter. This formula can be used as a basis for

estimating the value of business income that could be terminated at some point in the future, due to an unexpected event that has a low chance of occurring.

The expected net present value of a stream of income π, with profit in year one π_1, discount rate r, and the periodic growth rate g, for the time periods $t = 1, \ldots, \infty$, where the income stream is subject to termination risk governed by a Poisson process, is:[57]

$$E(\text{NPV}) = \frac{\pi_1}{r - g + \lambda}, \tag{1}$$

where λ = mean arrival rate of a Poisson process, or the chance that the event will occur in any one period.[58] This formula is deceptively simple; the familiar "Gordon Growth Model" equation (for perpetual, constantly growing cash flows without uncertainty) is:

$$\text{NPV} = \frac{\pi_1}{r - g}. \tag{2}$$

Thus, the effect of an event that would end the stream of income, and has a 2% chance of occurring in any one year, is similar to increasing the discount rate on the stream of income by 2%.[59]

4.8. Uses for Jump Processes: Franchised Firms

Many valuation tasks involved franchised firms, including both the manufacturers (typically the "franchisor") and the retailers or distributors (typically the "franchisee"). Even for well-developed franchises there is a nonnegligible risk that the franchise will fail, meaning that the products and services sold under the brand will no longer be produced or sold. In such cases, some franchised firms would suffer catastrophic losses and may cease doing business.

Anderson (2004a) suggests an "Oldsmobile rule" for estimating the specific brand risk for franchised firms. He notes that even the most firmly established brands eventually go out of business, and cites the Oldsmobile brand as establishing the upper limit of the likely tenure for most brands. Oldsmobile automobiles were continuously produced for 100 years before the brand was terminated by General Motors.

Anderson states that franchised firms bear a brand risk that can be modeled as a Poisson distribution. He then characterizes the brand risk in many franchised firms as a Poisson process with a mean arrival rate equal to 1/100 or higher.[60] A mean arrival rate of 1/100 means that one expects the termination of the brand to occur about once every century. Firms with weaker brands – and most franchises will be weaker than Oldsmobile – should be modeled with a higher mean arrival rate.

One implication of this research is that firms with significant, specific income termination risks are improperly modeled with a standard CAPM-based discount rate. For example, consider a "beta" derived from a study of large publicly traded firms in a certain industry, none of which are dependent on a single patent, franchise, supplier, customer, or market. Now consider a firm in the same industry that has earnings subject to termination risk, albeit a small risk in any one year. Many practitioners would simply use the beta for the industry, perhaps modify it for the size of the firm, and estimate a discount rate using a CAPM-derived formula. Such an approach would probably result in an overestimate of the market value of the firm, stemming from ignoring the risk of termination of income. Indeed, Monte Carlo testing of the formula shown in the first equation, using a very low mean arrival rate, confirms that it properly estimates the lower net present values that will be generated by cash flows that are subject to small risks of termination in any one year.[61]

4.9. Dynamic Programming Valuation Method

A method of solving complicated, multi-stage optimization problems called *dynamic programming* was originated by American mathematician Richard Bellman in 1957.[62] Like most brilliant insights, the method promised a radical simplification of some intransigent problem. However, the method was usually difficult, or even impossible, to implement in practice until quite recently. Developments in both analytical and computational methods now make it possible to use this method in business valuation. The use of dynamic programming for business valuation was introduced by Anderson (2004b). We very briefly summarize it here as a method that, while novel in practice, has great potential to improve valuation in the future.

The essence of this approach is Bellman's insight that an optimization problem can be segmented into two parts: the current benefit (the return on an investment in the current period) and the change in the value at the end of this period (the change in the discounted future benefits). This is,

for business valuation, analogous to estimating the value of a firm that originates from two parts:

1. The income expected in the current period; and
2. The value of the firm at the beginning of the next period, taking into account the prospects for future earnings at that point.

Note that many discounted cash flow schedules represent data in a similar way: the first columns show income during the next few periods, during which the income can be explicitly forecasted, and then the last column shows a "terminal value" which is the expected value at that time. However, there is a key difference between the approaches: the dynamic programming approach requires the management to *optimize* the sum of the value arising from current-period income and future-period expected earnings. Thus, in contrast to the income statements common to valuation projections,[63] the dynamic programming method assumes that managers will change expenditures when revenues change. Furthermore, it does not implicitly assume that the growth rate for revenue, or ratio of expenses to revenue (even for "variable" expenses) will remain the same.

4.10. Advantages of the Dynamic Programming Approach

One key advantage of this method is its proper assertion of the primary importance of management policy. Traditional valuation methods often assume a very passive role for managers.[64] In contrast, dynamic programming puts management policy front-and-center. It is the manager who is assumed to *optimize* the sum of current earnings and discounted future value. Owners of a firm, of course, have the same such incentives. The mathematics of solving dynamic programming models, while arcane and difficult, at least have the cardinal virtue of mimicking the actual incentives of business owners. This is not true in typical discounted cash flow models; management is often assumed away, becoming almost unimportant. On the other hand, dynamic programming assumes that the manager (or owner) optimizes the sum of current earnings and future discounted earnings, matching the mathematical techniques to the actual motivation of owners.

Thus, dynamic programming methods should, *ceteris paribus*, lead to better valuation estimates, as they explicitly consider policy actions that are largely ignored in traditional cash-flow models. Underlying this assertion of management importance is a superior treatment of uncertainty. The dynamic programming method incorporates the notion of the management of a firm changing its policies as business conditions change. By contrast, most discounted cash flow schedules described in the texts cited as references assume passive management and stable business conditions.

We briefly describe the technique in the Mathematical Appendix. A recent description of potential practical uses in Anderson (2005).

4.11. Disadvantages of the Dynamic Programming Method

While this method has important theoretical advantages, it suffers at this time from significant practical disadvantages. In particular, it has just been introduced, there are yet no practical software tools, and there are few texts on its use in this field. We expect this to change, however, and suggest that practitioners interested in cutting-edge techniques anticipate much development in this technique.

5. LEGAL DEVELOPMENTS

Since the seminal decisions in *Daubert* and *Kumho Tire*, courts have been systematically restricting the use of expert testimony through the application of what are commonly called "Daubert standards."[65] These standards, given the relative newness of the seminal decisions, are more like an evolving consensus than a clear universal standard.

Given the vital importance of this issue, we describe below the most important developments for litigation economics covering business valuation in the evolving Daubert standards. For a broader summary, see Gaughan (2003).

5.1. Daubert Standards Apply in Business Valuation

One of the most important decisions in this area was *Ullman-Briggs, Inc. v. Salton/Maxim Housewares*, Inc., 1996 WL 535083, (N.D. Ill. 1996). In this case, the court applied the Daubert standards to business valuation, stating:

> While business valuation may not be one of the "traditional sciences," it is nevertheless a subject area that employs specific methodologies and publishes peer-reviewed journals.

The court rejected the request of the plaintiff in this case to offer as "expert" testimony in business valuation a person who was a business broker and

who provided an estimate of the value of firm based largely on his experience in selling businesses. The court concluded that "an expert that supplies nothing but a bottom line supplies nothing to the judicial process."[66] The *Ullman-Briggs* decision is already influencing other courts, as will be seen in the discussion below.

5.2. Economics Experts Must Show Methodology and Data

Recent court cases have affirmed the common-sense rule that the expert's report could only be given credence if it included both the methodology used to arrive at the decision and the information on which the decision is based. Rules 26 and 702 of the Federal Rules of Civil Procedure already require this (for federal cases), but courts are beginning to enforce it with some vigor in cases involving business economics.

Indeed, a recent brief by the U.S. Department of Justice in *United States v. First Data Corporation* provides a rendition of the many possible legal attacks against a report given by a person of some business expertise, but whose conclusion was produced without adequate discussion of methodology and data. This anti-trust case turns to some degree on the question of whether two different payments methods are competing products. A business executive with no economics training produced an opinion on this question, which the Department of Justice moved to exclude with a withering series of arguments:

> It is well established that an expert witness must have a grounding in the methods and procedures of a particular field, and that expertise must be applied in a way that enables the witness to draw conclusions about the particular issues in the case. See Daubert, 509 U.S. at 590-91. These requirements are not abandoned when a witness attempts to rely solely or primarily on experience as a basis for non-scientific opinions. Under those circumstances, "[t]he trial court's gatekeeping function requires more than simply 'taking the expert's word for it.'" Fed. R. Evid. 702 2000 Advisory Committee Notes. Instead, the court must require the witness to explain "how that experience leads to the conclusion reached, why that experience is a sufficient basis for the opinion, and how that experience is reliably applied to the facts." Id. See, e.g., *United States v. Jones*, 107 F.3d 1147 (6th Cir. 1997) (Handwriting examiner, who had years of practical experience and extensive training, explained his methodology in detail.) Indeed, the advisory committee notes to Rule 702 quote with approval the Fifth Circuit's admonition that "[i]t seems exactly backwards that experts who purport to rely on general engineering principles and practical experience might escape screening by the district court simply by stating that their conclusions were not reached by any particular method or technique." *Watkins v. Telsmith, Inc.*, 121 F.3d 984, 991 (5th Cir. 1997).
>
> "[N]othing in either *Daubert* or the Federal Rules of Evidence requires a district court to admit opinion evidence that is connected to existing data only by the *ipse dixit* of the expert. A court may conclude that there is simply too great an analytical gap between the data and the opinion proffered." *General Elec. Co. v. Joiner*, 522 U.S. 136, 146 (1997). Because there is no methodological link between [the proposed expert's] industry experience and his opinions, his testimony fails to meet the standards of Rule 702.[67]
>
> ...It is elementary that an expert cannot simply point to his resume and then engage in unfettered speculation. Similarly, a witness with industry experience cannot just offer a "hunch" based on his business sense. *Ullman-Briggs, Inc. v. Salton/ Maxim Housewares, Inc.*, 1996 WL 535083 (N.D. Ill. 1996). As noted above, "[t]he trial court's gatekeeping function requires more than simply 'taking the expert's word for it.'"

The brief in this case is important for several reasons: first, it summarizes (from the perspective of the party attempting to restrict an expert's report) the recent relevant cases and federal rules governing this issue within the field of business economics. Second, the plaintiffs in the case include not only the U.S. Department of Justice, but also the attorneys general of six states and the corporation counsel of the District of Columbia. It therefore also represents a repository of legal arguments that will be widely shared. However, readers should note that it is a brief, not a decision.

There have been other cases that support this trend toward increasing scrutiny of experts. The California Appeals Court affirmed the exclusion of one expert's testimony after the expert failed to provide meaningful responses to valuation methodology questions in deposition.[68] The Arkansas Court of Appeals affirmed the exclusion of another expert's testimony, who had the credentials of a "certified financial analyst" but had no experience in business valuation.[69] The court, instead accepted the testimony of a court-appointed business appraiser using two different methods.[70]

5.3. General Business Experience Does Not Qualify as Economic Expertise

The same Department of Justice motion cited above contains a valuable summary of the arguments that can be made to exclude the testimony of a person whose expertise comes largely from general business experience in the industry, but who has no specific economics training.

> A court should "exclude proffered expert testimony if the subject of the testimony lies outside the witness's area of expertise." 4 Weinstein's Fed. Evid. § 702.06[1], at 702–52 (2000). Although [proposed expert] has experience in the "payments industry"..., he lacks any education or training in economics or industrial organization. [He] thus does not have the requisite training or experience to determine whether PIN debit and signature debit are in the same product market.
>
> General industry experience does not qualify a witness to conduct the analysis required to define a product market for purposes of an antitrust case, and [the proposed expert] is

no more qualified to testify about relevant markets than other non-economist witnesses who have been precluded from offering such testimony in similar circumstances.

In *Berlyn v. Gazette Newspapers*, 214 F. Supp. 2d 530, 536 (D. Md. 2002), for example, the plaintiffs' proposed expert witness had considerable experience in publishing, having held several prominent positions with newspapers throughout his career. Id. at 533. Nonetheless, the court determined that the witness was not qualified to opine that the relevant product market was community newspapers and some editions of metropolitan newspapers because the witness's background was "completely devoid of specific education, training or experience in economics or antitrust analysis." Id.; see also id. at 536 ("[G]eneral business experience unrelated to antitrust economics does not render a witness qualified to offer an opinion on complicated antitrust issues such as defining relevant markets").

Similarly, in *Virginia Vermiculite, Ltd. v. W.R. Grace & Co.*, 98 F. Supp.2d 729 (W.D. Va. 2000), the court prevented a geological engineer with some background in economics and substantial mineral industry experience (including experience performing market analyses for clients) from testifying as an expert about the geographic market for vermiculite. Id. at 732–734. The court noted that "there are differences between an analysis for business investment and an analysis for antitrust purposes," that "market analyses for antitrust markets generally require some expertise in the field of industrial organization," and that individuals with experience in analyzing the mineral market but not in antitrust "would not possess the skill and training of a professional economist necessary to define a relevant market for antitrust purposes."[71]

As stated above, an economic analysis of the industry, rather than just accounting records, must be used to estimate future earnings. In certain areas of law, specific economic analyses are required. However, such economic analyses need not be performed by an economist.

A Federal Court of Appeals decision in the *Ericsson v. Harris* patent infringement case highlights this allowance. The Court's review of the law was that to support a finding of lost profits, the relevant markets must be analyzed and a causal link found between the infringement and the lost sales. The Court then stated:

> Such market reconstruction must be supported by "sound economic proof of the nature of the market and likely outcomes with infringement factored out of the economic picture."[72]

Despite this straightforward requirement for economic analysis, the plaintiff's expert on damages was an accountant. The expert presented detailed market analyses, which included a segmentation of the relevant markets, consideration of barriers to entry, and actual sales records. The opposing counsel attacked the analysis as lacking a specific economic test for cross-elasticity of demand between two products.

However, the court accepted the argument that the analysis was developed "using an approved methodology" and was "supported by testimonial and documentary evidence."[73] Therefore, the Court affirmed that "substantial evidence supports the jury's damages award for lost profits due to lost sales."[74]

5.4. Lost Profits in Franchised Businesses

A growing number of businesses operate under franchise agreements, which allocate to the franchisor (usually the manufacturer) the right to establish a distinctive brand and set forth advertising, quality, and other standards, while the franchisee actually sells or distributes the products. The question of damages arises naturally in cases involving a claim of improper termination of the franchise, and a small body of case law has developed in this area. Important cases in this area were summarized in Fitzgerald and Anderson (2004).

A terminated franchisee's remedy for improper termination is generally money damages, not the continued use of the franchisor's brand.[75] The franchisee must present sufficient evidence of the damages; and if lost profits are claimed, they must be proved with reasonable, though not exact, certainty.[76] When there is a sufficient history of sales, a "before and after" comparison can be used to establish damages.[77] State courts have consistently found that improperly terminated franchisees are eligible for damages for lost future profits.[78] In addition, a number of state and federal laws provide specific protections to franchisees.[79]

5.5. Conclusion: Case Law on Valuation and Damages

From this survey we can observe a handful of themes in recent case law. These include:

- Continued development of the "Daubert" standards for expert testimony.
- Requirement for actual economic knowledge of markets or business valuation methodology, rather than simple credentials or "business experience".
- Awareness of the nature of franchises.
- Requirements for data and methodology to support a valuation or damages estimate.

NOTES

1. Complete citations of these and other sources are given in the reference list at the end of this chapter. Where more than one edition of a text is in common use, we have noted both in this section.

2. These include Reilly and Schwiess (1999); Copeland, Koller, and Murrin (2000), and others.

3. This contrasts strongly with the literature on financial assets viewed as part of an investor's portfolio, which is almost entirely based on economics, finance, and mathematics. See, e.g., Cochrane (2001); Duffie (2001); Markowitz (1991), or LeRoy (1991).

4. Of course, they typically summarize financial market research, such as CAPM models, that were parameterized by linear regression or other statistical techniques. However, such analysis is typically assumed to have been done by others.

5. A survey question posed by the author on the bulletin board of the National Association of Forensic Economics in July 2004 elicited about 20 responses, almost all of which indicated heavy use of spreadsheet software for business valuation or lost earnings for individuals analysis. Only a fraction (on the order of magnitude of 10%) indicated regular use of statistical, mathematical, or simulation software.

6. Many of the references listed below provide background for the definition; see, in particular, Hitchner (2003) and Anderson (2004b).

7. We do not discuss here the use of this approach when determining the salvage value of hard assets, such as in a liquidation or bankruptcy case.

8. The first statement of accounting is generally credited to Luca Pacioli (1447–1517), a Franciscan monk and mathematician in what is now Italy. He published *Summa de Arithmetica, Geometrica, Proportioni et Proportionalite* in 1494, summarizing mathematical knowledge of the time. See Anderson (2004b), Chapter 11.

9. See, e.g., Larson and Miller (1993, Chapter 1), stating the "cost principle" in the 13th edition of their accounting text. This is consistent with their definition of accounting: "the function of accounting is to provide useful information to people who make rational investment, credit, and similar decisions," quoting the Financial Accounting Standards Board, "Statement of Financial Accounting Concepts Number 1," (1978) paragraph 34.

10. See Adam Smith, *Wealth of Nations* (1776), Book I, Chapter 6. A review here must also note the contributions of David Ricardo (on the labor theory of value) and Alfred Marshall (on the introduction of the ubiquitous supply demand curves to determine price). Karl Marx and his followers have developed a literature devoted to the study of the differences between value and price, almost all of which is without practical use in this field.

11. See, e.g., Duffie (2001) for a mathematically rigorous reference of asset valuation in complete markets, based on the avoidance of arbitrage. Many other mathematical economics references, such as Ljungvist and Sargent (2004), contain summaries of this approach.

12. Treasury Department Appeals and Review Memorandum 34 ("ARM 34"), 1920. See Hitchner (2003, Chapter 4).

13. IRS Revenue Ruling 68–609.

14. In particular, the 1920 observation about the sources of equity returns predates the development of modern portfolio theory by approximately 40 years. Considering that most analyses of historical returns *start* with data from after 1920, one can gain an appreciation of how ahead of its time it was.

15. Among other problems, it is intended only to value *intangible* assets, but is often misused to value entire businesses; proper use requires the use of market value estimates of tangible assets, but the book values are often used; it is unclear how the intangible value of a firm's assets can be estimated using the known market value of its tangible assets, given that the two are probably intertwined; and the tricky use of different capitalization rates is bound to produce frequent errors.

16. Indeed, Pratt (2002, Chapter 17) indicates that the method is usually described in a full chapter of valuation references, citing both the third edition of his widely used text and the fourth edition (Pratt et al., 1996, 2000), as well as Fishman, Pratt, Griffith, and Wells (2001). He then proceeds to detail a "sanity check" for the calculation and offers a number of comments on the "vagaries" of its use.

17. There is some ambiguity and confusion about the nomenclature here; some authors use "capitalized income" to refer to a family of models that discount (capitalize) future earnings; others use "capitalized income" to only refer to a subset of the income approach in which a constant stream or earnings are discounted by a constant discount rate. There is even confusion in the literature on whether "discounted cash flow" is, in any sense, equivalent to "capitalized income." Part of this stems from the accounting convention that defines "cash flow" differently than "earnings."

However, from the point of view of the investor (not the CFO of the company itself) *distributed* cash to shareholders is the return on the ownership interest, and would commonly be considered "income." Indeed, the "capitalized income" that arises from dividends on a stock is theoretically its market value. (This arises from an absence of arbitrage, as well as the time value of money.) The derivation of the present value formula for a perpetual constant series of cash flows ($pv = cf/r$) typically arises from an assumption of a constant flow of future dividends. See, e.g., LeRoy (1991); Anderson (2004b).

Note that the dividend-paying firm's accounting cash flow, like other accounting concepts, never enters into the formula. In this sense, "discounted cash flow" is only equivalent to market value when, again under certain conditions, it is equivalent to capitalized income.

18. All the references in the "Standard Approaches" section describe these methods. The discussion there highlights the different emphases placed by different authors on certain aspects of the methods.

19. Although the nomenclature varies, these methods are often called "free cash flow to equity" and "free cash flow to the firm." Here "free cash flow" (or "net cash flow") is the distributable cash to investors or to the firm as a whole.

20. This assumption is often incorrect on both grounds: last period's earnings will usually not closely predict the earnings under new management or ownership, and the growth rate of a firm never stays constant. This latter assumption is later treated more closely in the sections titled "Methodological Developments" as well as in "Evidence: Past Earnings Not Sufficient Predictor."

21. The author has seen this error carried to its logical extreme. In one valuation estimate he reviewed, a distribution firm whose franchise had been *terminated for*

cause was projected to have earnings blithely continue for years at a constant growth rate. The counter analysis was fairly simple: forecasted earnings of zero continuing perpetually.

22. Hitchner (2003) contains the most extensive discussion of RR 59–60. He counts eight categories in RR 59–60, namely: (a) nature and history of business, (b) economic outlook for industry, (c) book value of the stock, (d) earning capacity, (e) dividend capacity, (f) whether the firm has goodwill or intangible value, (g) sales of stock, and (h) market price of stock in similar business ("Selected Revenue Rulings," reprinting RR 59–60, Section 4.01 (a)–(h)).

See also Gaughan (2004) for a corroborating opinion and Anderson (2004b) for a new synthesis of the factors required by the IRS.

23. Damodaran (1996, Chapter 7) is a good example. The importance of properly forecasting income using the industrial and economic fundamentals is made quite clear to a thorough reader, who would also discover empirical evidence confirming the failure of naive constant growth estimates. However, the methods described (time-series models, and averages of historical rates) are largely based on extrapolating past growth. Thus, for many readers, extrapolating past growth seems to be the easiest path.

24. Two of the recent accounting-perspective references also have an increased emphasis on proper forecasting of income. Abrams (2001, Chapter 2) places the forecasting of revenues and expenses at the beginning of his treatise and describes the use of regression analysis using independent variables (such as GDP) to help identify the relationship between the firm's earnings and the economy as a whole. Hitchner (2003, Chapter 2) explicitly instructs analysts to obtain "external information" regarding the future earnings of the firm.

25. The classic references include Markowitz (1952), Markowitz (1970), and Sharpe (1964). An excellent summary is also contained in Markowitz (1991).

26. This derivation is described most extensively in Damodaran (1996), although all the references in the "Standard Approaches" section contain discussions.

27. Among these are the Fama-French 3-factor model and the Arbitrage Pricing Theory. In the "Methodological Developments" section, we will discuss a simple log-size model and other discount rate innovations.

28. We describe below one specific risk faced by franchised businesses: termination of franchise rights. See "Modeling Income Subject to Termination Risk" in the appendix.

29. Among the general valuation references, Pratt et al., (1996) and Hitchner (2003) have the most comprehensive discussions.

30. *Business Valuation Discount and Premiums*, Pratt (2001).

31. By "nonhomogeneous" we mean assets of a different type that cannot be easily combined in an enterprise.

32. The cost-of-capital factors here may be appropriate to recognize as a discount or premium in the cost of (equity) capital. The cost of debt should already be adjusted to the firm's specific characteristics.

33. In practice, it may be closer to one-and-a-half-times discounting, which is bad enough.

34. Pratt (2001, Chapter 20) lists the "indiscriminate use of average discounts or premiums" as a common error. Hitchner (2003, Chapter 8) states that "the blind

application of discounts, without a thorough understanding of the subject company as compared to the data underlying the discounts, can lead to misleading valuation results." He further warns against "double discounts" caused by applying two discount factors arising from the same characteristic.

35. For a summary of the standard analyses of the size effect – which are sometimes interpreted as evidence that the CAPM model is flawed – see Damodaran (1996). Pratt (2001, Chapter 11) devotes an entire chapter to the size effect.

36. In the (normal) case where the cost of debt is lower than the cost of equity, understating the equity portion will result in an artificially low WACC estimate, resulting in an artificially high valuation estimate.

37. Among the valuation texts cited above, Anderson (2004b) and Damodaran (2002) explicitly cover it. Other texts often describe option valuation as an offshoot of the Black-Scholes model for valuing certain financial options, but ignore the much more common option to wait to purchase most investments.

38. Of course, the expected net present value of a dollar bet in a casino is *less* than a dollar. Therefore, it is the casino owners that routinely follow the rule!

39. See, e.g., Brealey and Meyers (1981), which was the first edition of the long-running corporate finance book. See also the additional citation from the fourth edition of this line of textbooks, below. The text was released in a seventh edition in 2003.

40. Reilly (1994), quoted in Pratt et al., (1996).

41. Brealey and Myers (1992), quoted in Pratt et al., (1996, Chapter 9).

42. See, in particular, Anderson (2004b), which models different types of uncertainty and directly incorporates managerial options in the discussion of valuation; and Schwartz and Trigeorgis (2001), whose compilation of important articles on this subject provide ample theoretical foundation for the insights on the value of managerial flexibility.

Among the standard valuation texts, Damodaran (1996) has extensive discussions of "contingent claims" analysis which rely on this insight. See the section on "The Real Option" in the next section. Damodaran (2002) covers the option to wait explicitly.

43. The original article was Black and Scholes (1973).

44. There are many references to financial options and methods to value them; most valuation references instead include a basic summary of call and put options for use in discussing contingent claims. Those interested in financial options should consult one of the texts devoted exclusively to this topic.

45. Damodaran (1996) goes as far as calling the "contingent claim" approach one of the three main approaches; he combines the market and asset methods into a "relative value" approach and retains the capitalized income approach.

46. In particular, the standard "beta" is a measure of the covariance of the returns from firms in one industry with that of the market as a whole. The equity premium is an average of the excess returns for equities above risk-free securities. Neither of these are the same as the contingent claim risk on one firm.

47. An "in the money" option is one that has an intrinsic value at the current time, such as a call option on a stock that is trading above the strike price. An option on a stock that is quite volatile, even if the current price is below the strike price, will tend to be valued more highly than one that is expected, quite steadily, to remain "out of the money."

48. Most importantly, a discounted cash flow analysis of a profitable operating business would be the analogy of a financial call option that was deeply "in the money."

49. See the section on "Dynamic Programming Valuation Method" and the references there, particularly Dixit and Pindyck (1994).

50. See the earlier section on "Critique: Inadequate Analysis of Forecasted Income."

51. Indeed, it is common to see forecasted earnings (or revenue) grow at a constant rate with no deviation. Some texts describe two-stage or three-stage forecasts, in which growth rates remain smooth but decline after an initial set of high-growth periods. In both these cases, there is little or no incorporation of uncertainty in the forecasted revenue.

52. See, e.g., Duffie (2001), one of the standard graduate texts in finance.

53. For a broad survey of advanced methods, which includes management firms as well as equity research and trading firms, see Facardi and Jonas (1997). This survey shows the intellectual ferment in areas that appear to be disconnected from the standard discounted cash flow methods in the (professional) valuation literature.

54. The "method of moments" is a very old method in statistics. The mean is the first moment of the distribution about zero, and the variance is the second moment about the mean.

55. An appeal to the Central Limit Theorem of Statistics is often made to justify an assumption of normally distributed disturbances. This may be justified if there are a very large number of small risk factors that are uncorrelated. Such an assumption is close to correct when dealing with a large, publicly traded firms.

See Anderson (2004b, Chapter 10) for a discussion of various types of uncertainty in revenues, starting with variation around a trend and continuing through its processes and geometric Brownian motion.

56. These risks in the United States are normally considered negligible. However, especially in the areas of litigation risk and environmental liability, they are often not negligible even in the U.S.

57. This discussion is based on Anderson (2004a). See also Anderson (2004b, Chapter 10). The basic derivation of the NPV formula for a constant series governed by Poisson risk is in Dixit and Pindyck (1994). A more rigorous mathematical discussion is in Duffie (2001).

58. More precisely, the "mean arrival rate" is the average number of events that occur each period over a large number of periods. The Greek letter *lambda* is often used to indicate this parameter.

59. We say "the effect...is similar" because the formula can be simplified into a form similar to the Gordon Growth model equation. The underlying cash flows are not at all similar, and one calculates an expectation while the other calculates a certainty. See the discussion below.

60. See Anderson (2004a).

61. Anderson (2004a) used a mean arrival rate of 1/100, periods of 100 years, a discount rate of 8%, and a growth rate of 4%. Over 100 trials, the formula and the average of the randomly-generated trials were fairly close, while ignoring the termination risk produced a significant overestimate of the net present value.

62. Bellman (1957). Bellman's introduction to his slim book of nearly 50 years ago is still useful reading.

63. The common presentation of discounted cash flow schedules is a projection of income growing at a constant rate, along with variable and fixed expenses growing also at constant rates, with no deviation around the trend. See, e.g., most of the discounted cash flow examples in the accounting-based literature listed in the "Standard Approaches" section.

64. Indeed, our critique of insufficient analysis of earnings forecasts above indicates that some valuation estimates literally assume *no* role for management; they simply extrapolate future earnings from current earnings, as if they would magically appear no matter who minds the store.

65. *Daubert v. Merrell Dow* 509 U.S. S. Ct. 579 (1993); *Kumho Tire v. Carmichael*, No. 97-1709, 526 U.S. 137, 119 S. Ct. 1167 (1999).

66. See the discussion of this case in Gaughan (2003, Introduction).

67. Memorandum of law in support of motion to exclude witness, *United States v. First Data Corporation and Concord EFS Inc.*, U.S. District Court for the District of Columbia, 2003. Because this is a memorandum, not a decision; because the reply brief was not available; and because the purpose for quoting the memorandum extensively was to include a number of potential legal challenges to expert witnesses in general; I have excluded the name of the witness. It is provided for reasons noted in the text, and not as the final decision in this case, or as a balanced summary of all cases discussing this issue.

68. *Phase 2 Developers Corp v. Citicorp Real Estate*, no. B160111, Cal. App. 2d Dist, (2004), unpublished. Summarized in Stockdale (2004).

69. *Thomas Sanders v. Heidi Sanders*, no. 03-738, Ark. App. (2004), unpublished. Summarized in Stockdale (2004).

70. The summary, cited in the note above, indicates that the appraiser used both the excess earnings and discretionary cash-flow method to determine that there was no "salable goodwill" in the business. As noted above, the excess earnings method is of doubtful credibility, but at least was intended to value intangible property. If backed up by a sound discounted cash flow analysis, a finding of no goodwill value in a business would probably be sound.

71. Memorandum of law in support of motion to exclude witness, *United States v. First Data Corporation and Concord EFS Inc.*, U.S. District Court for the District of Columbia, 2003. See cautionary note above and in the text.

72. *Ericsson et. al. v. Harris Corporation et. al.*, U.S. Court of Appeals for the Federal Circuit, no. 02-1571, -1603 (2003); Section B.2. The quoted section of the excerpt is from *Grain Processing Corp. v. Am. Maize-Prods. Co.*, 185 F.3d 1341, 1350 (Fed. Cir. 1999).

73. The methodology was asserted to be set forth in *Panduit Corp. v. Stahlin Bros. Fibre Works, Inc.*, 575 F.2d 1152 (6th Cir. 1978),

74. *Ericsson*, cited above, B.2.

75. *Burger King v. Jajeed*, 805 F. Supp. 994, 6 FLW Fed D 481 (SD Fl, 1992).

76. See Blair (1988); *Atlantic Sports Boat Sales v. Cigarette Racing Team*, 695 F. Supp. 58 (Mass, 1988); *Lindevig v. Dairy Equipment Co.* 150 Wis 2d 731, 442 NW2d 504 (1989).

77. *C.A. May Marine Supply Co. v. Brunswick Corp.*, 64g F.2d 104g (5th circuit, 1981).

78. Fitzgerald and Anderson (2004) list cases from California, Florida, Illinois, Indiana, Iowa, Minnesota, Missouri, Montana, Nebraska, New Jersey, Pennsylvania, Puerto Rico, South Carolina, and South Dakota, as well as federal court decisions.

79. Automobile dealers and beer and wine wholesalers are often covered by specific state statutes; there are also federal statutes for service station operators and auto dealers.

80. This has implications in many fields, including finance. In particular, if there are not a wide variety of securities that create a risk-reward frontier or that could, under additional assumptions, be expected to have normally distributed returns or risk characteristics, many of the nice, standard conclusions of modern portfolio theory are undermined.

81. The probability density function shows that the third equation can produce non-integer numbers, but these are probabilities of certain integer values, not the values themselves. Random Poisson numbers always produce integers. For example, a Monte Carlo run of random Poisson numbers with $\lambda = 0.25$ produced 100 numbers: mostly zeroes, some ones, and one two. The sum of all 100 numbers was 21; 21/100 is close to the mean arrival rate of $\lambda = 0.25$.

82. More complete explanations of the dynamic programming method for optimization problems in economics and other fields are Miranda and Fackler (2002); Chiang (1999); Rustagi (1994), and Ljungqvist and Sargent (2004).

83. The "state" vector captures information about the state of the world at any one point. State variables can include those summarizing the business, the industry, or the economy as a whole.

84. This is best described in Rustagi (1994) and Chiang (1999). Chiang notes that this is not a mapping from real numbers to real numbers, which would be a function. Instead, it is a mapping from *paths* to real numbers, the real numbers being the quantities being optimized.

85. For the mathematical proofs that solutions are possible and unique, see Stokey and Lucas (1989).

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REFERENCES

Abrams, J. B. (2001). *Quantitative business valuation*. New York: McGraw-Hill.

Anderson, P. L. (2004a). Valuation and damages for franchised businesses. Paper presented at ASSA Conference, San Diego, California (January 2004). Available as AEG Working Paper no. 2003-12 at: http://www.andersoneconomicgroup.com.

Anderson, P. L. (2004b). *Business economics and finance: Using Matlab, simulation models, and GIS*. New York: CRC Press.

Anderson, P. L. (2005). Practical dynamic programming for business and forensic economics. Paper presented at the National Association of Forensic Economics international conference, Dublin, Ireland (May 2005). Available as AEG Working Paper no. 2005-5 at: http://www.andersoneconomicgroup.com.

Bellman, R. (1957). *Dynamic programming*. Princeton University Press; reissued at Mineola, New York by Dover Press, 2003.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political economy, 81*, 637–654.

Blair, R. D. (1988). Measuring damages for lost profits in franchise termination cases. *Franchise Law Journal, 8*, 3.

Brealey, R., & Myers, S. (2003 [1991] [1981]). *Principles of corporate finance*. New York: McGraw-Hill.

Chiang, A. C. (1999). *Elements of dynamic optimization*. Longrove, IL: Waveland Press.

Cochrane, J. H. (2001). *Asset pricing*. Princeton. NY: Princeton University Press.

Copeland, T., Koller, T., & Murrin, J. (2000). *Valuation: Measuring and managing the value of companies* (3rd ed.). New York: Wiley.

Damodaran, A. (2002 [1996]). *Investment valuation*. New York: Wiley (References in the text of this article are to the first edition).

Dixit, A., & Pindyck, R. (1994). *Investment under uncertainty*. New York: Princeton University Press.

Duffie, D. (2001). *Dynamic asset pricing theory* (3rd ed.). New York: Princeton University Press.

Facardi, S., & Jonas, C. (1997). *Modeling the market: New theories and techniques*. New Hope, PA: Frank J. Fabozzi Associates.

Fishman, J., Pratt, S., Griffith, J. C., & Wells, M. (2001). *Guide to business valuations* (12th ed.). Fort Worth, TX: Practitioners Publishing Company.

Fitzgerald, P. W., & Anderson, P. L. (2004). Valuation and damages for franchised businesses: Overview of important case law and a proposed new methodology for estimating damages. Paper presented at the international meeting of the National Association of Forensic Economics, Edinburgh, Scotland (May 2004).

Gaughan, P. A. (2003). *Measuring business interruption losses and other commercial damages*. New York: Wiley.

Hitchner, J. R. (Ed.) (2003). *Financial valuation: Applications and models*. New York: Wiley.

Larson, K., & Miller, P. (1993). *Fundamental accounting principles* (13th ed.). California: Irwin.

LeRoy, S. F. (1991). Present value. In: J. Eaton, M. Milgate & P. Newman (Eds), *The new Palgrave: A dictionary of economics*. New York: Stockton. Also in the slimmer volume *The new Palgrave: Finance*. New York: Stockton.

Little, I. M. D. (1962). Higgledy piggledy growth. *Oxford Institute of Economic & Statistics, 24*(4), 387–412. Cited in Damodaran (1996, Chapter 7).

Ljungqvist, L., & Sargent, T. J. (2004 [2000]). *Recursive macroeconomic theory* (2nd ed.), Massachusetts: MIT Press.

Markowitz, H. M. (1952). Portfolio selection. *Journal of Finance, 7*(1), 77–91.

Markowitz, H. M. (1970). *Portfolio selection: Efficient diversification of investments*. New Haven: Yale University Press (Reprinted New York: Wiley).

Markowitz, H. (1991). Mean variance analysis. In: *The new Palgrave: A dictionary of Economics*. New York and London: Stockton and MacMillan.

Miranda, M. J., & Fackler, P. L. (2002). *Applied computational economics and finance.* Massachusetts: MIT Press.
Pratt, S. (2001). *Business valuation discount and premiums.* New York: Wiley.
Pratt, S., Reilly, R., & Schwiess, R. (1996). *Valuing abusiness* (3rd ed.). New York: McGraw-Hill. A fourth edition was released in 2000. References in the text of this article are to the 3rd edition).
Pratt, S. P. (2002). *Cost of capital* (2nd ed.). New York: Wiley.
Reilly, F. K. (1994). *Investment analysis and portfolio management.* Hinsdale, IL: Dryden Press.
Reilly, R. F., & Schwiess, R. (1999). *Handbook of advanced business valuation.* New York: McGraw-Hill.
Rustagi, J. S. (1994). *Optimization techniques in statistics.* New York: Academic Press.
Schwartz, E., & Trigeorgis, L. (Eds) (2001). *Real options and investment under uncertainty: Classical readings and recent contributions.* Massachusetts: MIT Press.
Smith, A. (1776). *Wealth of Nations,* originally published 1776; Cannan's edition published (1907). Reissue Chicago: University of Chicago Press (1976).
Stockdale, J. J. (2004). Business valuation cases in brief. *Business Valuation Review, 47*(1).
Stokey, N., & Lucas, R. E., Jr. (1989). *Recursive methods in economic dynamics* (with Prescott, E. C.). Harvard, MA: Harvard University Press.
Wolpin, J. B. (2003). Examining the reliability of small business transaction databases. *Valuation Strategies, 8*(Nov–Dec), 24.

MATHEMATICAL APPENDIX

Modeling Income Subject to Termination Risk

We consider in this section random events that have a small probability of occurring in any one year. The normal distribution, or at least a nice symmetrical distribution of events, typically cannot be used to describe the risk of such events.[80] Instead, we suggest the *Poisson* distribution as an appropriate model for such risks. This statistical distribution is close to that of a binomial distribution in which the number of trials is very high and the probability of success in each trial is low. The binomial is often used to model events that have close to a 50% chance of occurring, such as a coin flip. The Poisson is typically used in studies of errors, breakdowns, queuing behavior, and other phenomena where the chance of any one subject facing a specific event is small, but where the number of subjects is large.

A Poisson process is governed by the following probability density function:

$$P(x) = \frac{e^{-\lambda}}{x!}\lambda^x, \quad x = 0, 1, 2, \ldots \tag{3}$$

note that the Poisson is a discrete probability distribution; it provides positive probabilities only for integers $x = 0, 1, 2, \ldots$.[81]

The Dynamic Programming Method

We present below a simplified description of the method of dynamic programming, originally developed by Bellman (1957) and recently applied to business valuation by Anderson (2004b).[82] In this derivation, we model the management of a business as a multi-period optimization problem.

1. A business is an organization which will live through multiple periods and for which a mixture of both reasonably predictable, and unknowable, events will occur. These events will present the management of the company with information, which can be summarized as data in a state vector.[83] At each time period, holding the information available, the management takes certain actions, such as hiring, firing, purchasing, pricing, advertising, and selling.
2. The challenge (the "optimization problem") presented to the managers of the company is to take actions in a manner that maximizes the value of the firm. If we take the value of the firm to be the expected future profits, discounted for time and risk, we can express this optimization problem in the following *functional equation:*

$$V(s, t) = \max_x \{f(s, x) + \beta E[V(s_{t+1}, x_{t+1})]\}. \tag{4}$$

Here, $V(s,t)$ is the value of the firm given the state s at the time period t. This value consists of two parts: the current profit of the firm $f(s, x)$ and the expected value of the firm in the *next* period, after discounting by the factor β. The discount factor is equivalent to $(1/1 + \rho)$, where ρ is the discount rate on the capital employed and is often considered to be around 15% in applied work.

The maximization problem involves the *control variables* or *actions* x, so the maximization operator references this variable or vector of variables. Because both the current profit and the future profits of the firm depend on the actions of the firm's management, the action variable is an argument to the profit and value functions.

The fourth equation is known as a *functional* equation because the expression $V(s,t)$ is not, strictly speaking, a function of just the variables s and t, but instead the maximization of a family of functions.[84] We will refer to it as the *Bellman equation* for this optimization problem.

Actually *solving* the problem we have now stated has been the greatest difficulty in using this technique.[85] Among the problems are the "curse of dimensionality," because the size of the problem is magnified exponentially by the number of variables; properly specifying the state variable; properly specifying the objective function; and the computational techniques.

There are a number of methods for solving dynamic programming problems, including:

a. Recursively solving the problem, backward, from a known terminal value; this is known as "backward recursion."
b. Iterating on the values created at each step with variations in the *policy* created by the application of the control variables; this is known as "policy iteration."
c. Specifying an initial set of values for all variables, calculating the value function at these points, and then iteratively searching for higher values until such searches yield no further improvement. This is known as "function iteration."

The mathematics underlying this approach, including a rigorous derivation of the conditions under which the technique can be expected to produce a unique solution, is outlined in Stokey and Lucas (1989). Ljungqvist and Sargent (2004) provide a set of applications to academic problems and notes on its applications to macroeconomics. A computational approach, along with a series of examples from the academic literature, was developed by Miranda and Fackler (2002). They also provide computer code that can be used with the vector-processing mathematical software environment Matlab. The application to business valuation and damages was developed by Anderson (2004b), with practical applications for both business valuation and lost wages presented in Anderson (2005).